



EUI WORKING PAPERS IN ECONOMICS

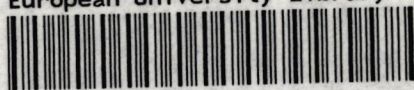
EUI Working Paper ECO No. 90/6

A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed

HAL R. VARIAN

European University Institute, Florence

European University Library



3 0001 0011 5458 4

Please note

As from January 1990 the EUI Working Paper Series is divided into six sub-series, each sub-series will be numbered individually (e.g. EUI Working Paper LAW No 90/1).

EUROPEAN UNIVERSITY INSTITUTE, FLORENCE
ECONOMICS DEPARTMENT



EUI Working Paper ECO No. 90/6

**A Solution to the Problem of Externalities
and Public Goods when Agents
are Well-Informed**

HAL R. VARIAN

BADIA FIESOLANA, SAN DOMENICO (FI)

All rights reserved.
No part of this paper may be reproduced in any form
without permission of the author.

© Hal R. Varian
Printed in Italy in April 1990
European University Institute
Badia Fiesolana
– 50016 San Domenico (FI) –
Italy

A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed.

by

Hal R. Varian
University of Michigan

November, 1989
Current version: February 14, 1990

Abstract. I consider economic environments involving externalities and public goods where agents have full information but the regulator does not. For these environments I present a class of simple two-stage games whose subgame perfect equilibria are efficient allocations. In the case of two-party externalities, the equilibria involve compensation for the party upon whom the externality is inflicted. In the case of public goods, the equilibria are Lindahl allocations.

Keywords. externalities, public goods, sequential games, Lindahl allocations, subgame perfect equilibria

Address. Hal R. Varian, Department of Economics, University of Michigan, Ann Arbor, MI 48109

A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed.

Hal R. Varian

Consider an economic environment in which some agents may take actions which impose benefits or costs on other agents. All agents are aware of the relevant technology and the tastes of all other agents. However the "regulator," who has the responsibility for determining the final allocation, does not have this information. How can the regulator design a mechanism so that the agents will have the proper incentives to reveal their information and achieve an efficient allocation?

In addition to implementing an efficient allocation, one might also want the mechanism to achieve some distributional goals. For example, one might want agents who are injured by an externality to be compensated for that injury. Or, in the case of public goods, one might want the public goods to be paid for by a system of Lindahl taxes.

In this paper I describe a simple two-stage game that implements efficient allocations in this sort of environment. The mechanism also achieves the distributional goals just described. In the subgame perfect equilibria of this game, parties injured by the externality are compensated and in the case of public goods, the mechanism implements Lindahl allocations. The mechanism appears to work in a broad variety of economic environments and does not involve substantial restrictions on tastes or technology. In addition, the mechanism is very simple to describe and analyze.

The fact that sequential games and subgame perfect equilibria may be very useful in implementation problems was first suggested by Moore and Repullo (1988). They show that in economic environments, almost any choice rule can be implemented by subgame perfect equilibria. However, as Moore and Repullo point out, "... the mechanisms we construct ... are far from simple; agents move simultaneously at each stage and their strategy

This work was supported by National Science Foundation grant SES-8800114. I wish to thank Mark Bagnoli, Ted Bergstrom, Ken Binmore, Arthur Robson, and Michael Whinston for their comments and suggestions.

sets are unconvincingly rich. We present such mechanisms to show what is possible, not what is realistic." (p. 1198)

However, Moore and Repullo also show that in certain "economic environments" it is possible to use somewhat simpler mechanisms. I pursue this idea and construct mechanisms that appear to be quite simple and practical for the sorts of the externality problems of interest to economists.

It should be emphasized that these mechanisms only work in the case where the agents are perfectly informed about the technology and tastes of the other agents. This is, of course, more restrictive than one would like. However, there is a broad set of cases for which such mechanisms may be useful. For example, consider a set of agents who must design a mechanism to make group decisions in situations that will arise in the future. At the time the mechanism is chosen, the agents do not know the relevant tastes and technologies, but they will know these things when the mechanism is actually used.

This latter example is sometimes known as a "constitutional choice" problem: we must choose a decision-making procedure without knowing much about the tastes and technologies available to the agents who will use the procedure. However, we do know that the agents who will actually use the procedure will be reasonably well-informed about these matters when the decision must be made. I suggest that the mechanism described below may be useful for such problems.

In section 1 I describe a very simple example of the mechanism in a two-firm externalities problem and discuss in an intuitive way why the method works. The following sections show how the method can be extended to work in more general environments. Next, I describe a generalization of the mechanism which implements Lindahl allocations in a standard public goods problem. Finally, I show how the method may be used to solve very general problems in resource allocation.

1. An example of the mechanism

Consider the following externality problem involving two firms. Firm 1 produces output x in order to maximize its profit

$$\pi_1 = rx - c(x).$$

Here r is the competitive price of output and the cost function $c(x)$ is a differentiable, positive, increasing, and convex function.

We assume that firm 1's choice of output imposes an externality on firm 2. For any choice of x , firm 2's profits are

$$\pi_2 = -e(x),$$

where $e(x)$ is a differentiable, positive, increasing, and convex function of x . All of this information is common knowledge among the agents, but is not known to the regulator.

One class of solutions to this externality problem involves re-assigning the property rights. For example, one firm could buy out the other, internalize the externality, and eliminate the inefficiency. Another class of solutions involves negotiation between the agents with respect to the externality. A third class of solutions involves intervention by a regulator who imposes a Pigouvian tax.

We will assume that the property rights are fixed, so that one firm cannot buy out the other or move away from the externality. Hence "property rights" solutions to the externality are not available and some type of negotiation between the firms or intervention by the regulator is necessary to encourage efficient outcomes.

If the regulator had full information the problem would be easy. One solution would be for the regulator to impose the costs of the externality on firm 1 by charging it a "tax" of $e(x)$ if it produces x units of output. Firm 1 would then solve the problem

$$\max_x rx - c(x) - e(x).$$

Let x^* be solution to this problem; then x^* satisfies the first-order condition

$$r - c'(x^*) - e'(x^*) = 0.$$

Given our curvature assumptions on $e(x)$, we could just as well set a "Pigouvian tax," $p^* = e'(x^*)$ and let firm 1 solve the problem

$$\max_x rx - c(x) - p^*x.$$

However, we have assumed that the regulator doesn't know the size of the externality and therefore cannot determine p^* . The regulator's problem is to design a mechanism that

will induce the agents to reveal their information about the magnitude of the externality and achieve an efficient level of production.

Here is a mechanism that solves the regulator's problem.

Announcement stage. Firm 1 and firm 2 simultaneously announce the magnitude of the appropriate Pigouvian tax; denote the announcement of firm 1 by p_1 and the announcement of firm 2 by p_2 .

Production stage. Firm 1 chooses x to maximize the following payoff

$$\Pi_1 = px - c(x) - p_2x - \alpha_1(p_1 - p_2)^2.$$

Firm 2 receives a payoff of

$$\Pi_2 = p_1x - e(x) - \alpha_2(p_2 - p_1)^2.$$

The parameters α_1 and α_2 are arbitrary positive numbers.

In this mechanism firm 1 is forced to pay a penalty based on the marginal social cost of the externality as reported by firm 2, and firm 2 receives compensation based on the marginal social cost as reported by firm 1. Each firm must also pay a penalty based on the square of the difference between the two reports. This penalty can be any increasing function of difference between the two reports, but we have chosen a quadratic penalty for simplicity. Since the α 's can be arbitrary positive constant, the penalties can be arbitrarily small.

Analysis of the mechanism

There are many Nash equilibria of this game; essentially any triple (p_1, p_2, x) such that $p_1 = p_2$ and x maximizes firm 1's objective function is a Nash equilibrium. However, if we use the stronger concept of subgame perfect equilibrium we get a much smaller set of equilibria. In fact, the *unique* subgame perfect equilibrium of this game has each agent reporting $p_1 = p_2 = p^*$ and firm 1 producing the efficient amount of output.

In order to verify this, we must work backwards through the game. We begin with the production stage. Firm 1 maximizes its profits, given the reports generated in stage 1, which implies that firm 1 will choose x to satisfy the first-order condition

$$r = c'(x) + p_2.$$

This determines the optimal choice, x , as a function of p_2 , which we denote by $x(p_2)$. Note that $x'(p_2) < 0$ —the higher the cost that firm 2 announces, the less firm 1 will want to produce.

We now examine the price-setting stage of the game. Consider first firm 1. If firm 1 believes that firm 2 will announce p_2 , then firm 1 will want to announce $p_1 = p_2$. This is clear since p_1 has no effect on firm 1's payoff except through the quadratic penalty.

Consider now firm 2's decision. Although firm 2's announcement has no *direct* effect on firm 2's profits, it does have an *indirect* effect through the influence of p_2 on firm 1's output choice in stage 2. Differentiating the profit function of firm 2 with respect to p_2 , we have

$$\Pi'_2(p_2) = [p_1 - e'(x)]x'(p_2) - 2\alpha_2(p_2 - p_1). \quad (1)$$

We have seen that $p_1 = p_2$ in equilibrium. Hence if firm 2 is at an *interior* equilibrium we must have $p_1 = e'(x)$.

The second derivative of firm 2's profit is

$$\Pi''_2(p_2) = [p_1 - e'(x)]x''(p_2) - x'(p_2)^2 e''(x) - 2\alpha_2 < 0.$$

Here we have used the fact that $p_1 = e'(x)$ and $x'(p_2) < 0$. Hence the second-order condition must be satisfied at any interior solution.

If $p_1 = e'(x)$, then the first order condition for production is

$$r = c'(x) + e'(x),$$

which is simply the condition for social optimality. Hence, the unique subgame perfect equilibrium to this game involves firm 1 producing the socially optimal amount of the externality. Note that this is a *strict* equilibrium in the sense that each firm's maximization problem achieves a strict maximum at this equilibrium.

The only thing remaining to check is that there are no boundary solutions. Suppose that p can range between zero and \bar{p} . Since $p_1 = p_2$ in any equilibrium, the derivative of firm 2's profit at $p_2 = 0$ is

$$\Pi'_2(0) = -e'(x)x'(0) > 0.$$

Similarly, at $p_2 = \bar{p}$, we have

$$\Pi'_2(\bar{p}) = [\bar{p} - e'(x)]x'(\bar{p}).$$

If \bar{p} is larger than the largest possible marginal externality cost, this derivative will certainly be negative.

Note that in this mechanism, firm 2 is compensated for the externality. Since the externality cost function is convex, firm 2 in fact is *overcompensated* for the externality, in the sense that the amount of money it collects exceeds the cost of the externality.

2. Discussion of the example

The intuition behind the mechanism is not particularly difficult. In the first stage each firm announces the (marginal) cost of the externality: firm 1 announces a cost that will be used to compensate firm 2, and firm 2 announces a cost that will be used to tax firm 1.

It is more or less obvious that firm 1 will never want to say the externality is not as costly as it really is, since then firm 2 will then want to set a large tax for firm 1 so as to make the level of production as small as possible. This will certainly harm firm 1.

Will firm 1 ever want to overstate the cost of the externality by announcing that the externality is larger than it really is? If it does, the firm 2 will be overcompensated on the margin and will want to set $p_2 = 0$ so that firm 1 will produce as much output as possible. But then firm 1 will have to pay a penalty based on the difference between their two reports. Hence firm 1 can only lose by exaggerating the size of the externality.

Collusion

We have seen that each firm independently has an incentive to reveal the truth in equilibrium. Furthermore, in equilibrium the government's budget balances in the sense that the compensation paid to firm 2 is just equal to the tax paid by firm 1. But out of equilibrium the budget will not necessarily balance. Is it possible that the two firms can collude in some way so as to exploit the regulator?

The sum of the profits of the two firms using the mechanism is:

$$(r + p_1 - p_2)x - c(x) - e(x) - (\alpha_1 + \alpha_2)(p_1 - p_2)^2. \quad (2)$$

Ignore the quadratic penalty term in (2) for the moment. Then without this term, the firms would like to set $p_1 = \bar{p}$ and $p_2 = 0$. That is, firm 1 would want to exaggerate the magnitude of the externality in order to encourage the regulator to pay a large compensation to firm 2. But if firm 2 gets overcompensated for the externality, it wants to report $p_2 = 0$ so as to encourage firm 1 to produce as much as possible.

However, this strategy involves making highly divergent reports. Can we use the penalty term to discourage such collusion?

The quadratic penalty is not very good for this purpose since it has a derivative of zero when $p_1 = p_2$. However an absolute value penalty works reasonably well. To see this, let the penalty term be given by $\alpha_i |p_1 - p_2|$. If we choose $\alpha_2 = 0$ the proof given earlier goes through: $p_1 = p_2$ is still the unique subgame perfect equilibrium of the game.

Suppose that we consider increasing p_1 and decreasing p_2 . Then the profits of the firm increase by x , and the penalty increases by α_1 . Certainly for large enough values of α_1 this will not be a profitable move for the coalition.

Strict equilibria and information costs

We have assumed that each firm has full information. It is natural to suppose that firm 2 knows the cost of the externality, but firm 1 may not know the magnitude of the costs it imposes, at least with certainty. However, let us suppose that firm 1 may learn the size of the externality by incurring some costs. The question is, will firm 1 have proper incentives to actually make this investment?

Without modeling the information acquisition in detail we cannot give a precise answer to this question. However, roughly speaking, it appears that the answer is yes. Since the efficient output is a strict equilibrium, firm 1 incurs a penalty if it announces $p_1 \neq p_2$. The size of this penalty depends on the magnitude of the α_1 . The larger the value of α_1 the more incentive firm 1 has to match firm 2's announcement. Hence, firm 2 does have an incentive to invest in information acquisition.

Income effects

The above example used payoff functions that are linear in money, so that the objective functions of the agents are quasilinear utility functions. This is reasonable in the case of firm behavior, but somewhat restrictive in the case of externalities involving consumers.

However, the mechanism can easily be generalized to include income effects. Since this is a special case of the public goods problem and general resource allocation problem presented later, we defer the argument until we treat those topics.

3. Relation to the literature

There is a broad literature that is concerned with the design of games to implement desired allocations. See Laffont and Maskin (1982) and Maskin (1985) for overviews of this literature. There are three crucial aspects in the mechanism described above: first, that agents report on the type of other agents, second, that there is a penalty based on the difference in the reports, and third, that we use subgame perfection as an equilibrium concept.

Each of these features has appeared in other mechanisms available in the literature, but, to my knowledge, they have never been used together in the same way that I use them. For example, Matsushima (1988) and Palfrey and Srivastava (1989) describe mechanism where agents report on each others type. Each of these mechanisms involves penalties when the types are different.

Moore and Repullo (1989) use all three devices in their basic mechanism, but the mechanism is very complex. It is however, designed for more general problems than simply correcting externalities. I discuss the relationship with Moore and Repullo in more detail below.

4. Public goods

The externality described above is a very simple one: there are only two agents involved. It might be supposed that if property rights can be freely transferred, or other sort of negotiations can be undertaken, then it would be possible to internalize simple two-party externalities of this sort. The vast literature on the Coase theorem discusses this point in detail.

But problems involving several agents are not so easily dismissed. If the externality is a public good, then transferring property rights may be difficult or impossible. And totally unstructured negotiations may be very complex or costly to implement. However, a generalization of our mechanism may still be used effectively. We loosely describe the mechanism here, and give a more detailed description in the next section.

In the case of public goods, the relevant parameter for describing the tastes of an agent is simply his Lindahl price—his marginal rate of substitution at a particular Pareto efficient allocation. In stage 1 of our game, each agent i will simultaneously announce a “price of the public good,” p_{ij} , for each other agent j . The “price” that agent j will actually face is the average of the prices chosen for him by the other agents.

In the second stage of the game, each agent j determines how much he wants to contribute to the public good based on his personalized price determined in the first stage of the game. The amount that each agent has to pay for his contribution will depend on his own contribution and on the total contributions by the other agents.

Agents recognize in the first stage that the prices that they set will influence the Nash equilibrium of the contribution game in the second stage. We will show that in the unique subgame perfect equilibrium, each agent will announce a set of Lindahl prices, and the contributions in the second stage will comprise the associated Lindahl allocation!

5. A simple mechanism for the public goods problem

There are n agents and two goods, a private good, x_i , and a public good, G . Agent i has a differentiable quasiconcave utility function $u_i(G, x_i)$ and initial wealth w_i . We assume that the demand function for the public good is a differentiable function of its price; see the appendix for discussion of this point.

A unit of the private good can be transformed into a unit of the public good on a one-for-one basis. Hence if each agent j sacrifices g_j of the private good, the utility of agent i will be $u_i(\sum_{j=1}^n g_j, w_i - g_i)$. This is not a serious restriction on the technology that generates the public good. If G is produced from the private good by a concave production function $h(x)$, then the utility function of agent i is $u_i(h(\sum_{j=1}^n g_j), w_i - g_i)$, which simply involves reinterpreting the original utility function.

A Lindahl allocation for this problem is a set of personalized prices (p_i^e) for the public good, such that the amount of the public good that each consumer i demands at his personalized price is equal to a Pareto efficient amount of the public good. Since a Pareto efficient amount of the public good satisfies the Samuelson condition that the sum of the marginal rates of substitution equals the marginal cost, we must have $\sum_{i=1}^n p_i^e = 1$.

Our mechanism for solving this public goods problem consists of two stages.

The price setting stage. Each agent i announces $n - 1$ numbers p_{ij} for all $j \neq i$. The personalized price facing agent j is the average of the numbers named by the other agents:

$$p_j = \frac{1}{n-1} \sum_{i \neq j} p_{ij}.$$

Let p denote the vector of prices (p_1, \dots, p_n) .

The contribution stage. Each agent i chooses a nonnegative amount g_i to contribute to the public good. If the total amount of contributions equals G , then agent i must pay an amount $p_i G + Q(p)$, where $Q(p) = \alpha(\sum_{i=1}^n p_i - 1)^2$ and α is an arbitrary (small) positive constant.

I claim that the *unique* subgame perfect equilibrium of this game will be a Lindahl allocation.

It is easy to show that the Lindahl allocation is an equilibrium of this game. To do this we only need to show that if every other agent announces the Lindahl prices, agent i can not increase his utility by announcing some other set of prices.

To prove this we first observe that since the Lindahl prices generate an efficient amount of the public good, G^e , since $\sum_{i=1}^n p_i^e = 1$. Hence, $Q(p) = \alpha(\sum_{i=1}^n p_i^e - 1)^2 = 0$. Suppose

that agent i , say, announces some numbers that change the price vector p^e to p' and that this change results in some possibly different amount of the public good, G' . Note that the price facing agent i does not change since agent i can only affect the prices facing the other agents.

We have

$$u_i(G^e, w_i - p_i^e G^e) \geq u_i(G', w_i - p_i^e G') \geq u_i(G', w_i - p_i^e G' - Q(p')).$$

The first inequality arises from the fact that we start with a Lindahl allocation so that agent i 's demand for the public good at his personalized price is the actual amount of the public good. The second inequality arises from the fact that $Q(p') \geq 0$. This shows that agent i is at least as well off announcing numbers that lead to the Lindahl prices (p_j^e) as any other numbers.

The proof that there are no other subgame perfect equilibria to the game is somewhat more complicated and is given in the next section. However, it is worthwhile giving an intuitive argument here to get a feel for the mechanism. In order to do this we make the strong assumption that each agent contributes a positive amount in the second stage of the game. This will not generally be true, and dealing with this difficulty is what generates the complications in the formal proof given below.

Let p_i be an arbitrary set of prices that do not sum to 1. In the first stage of the game agent i gets to influence agent j 's price, p_j . We will show that agent i can change p_j in a way that will increase agent i 's utility.

The derivative of agent i 's utility with respect to p_j is

$$\frac{\partial u_i}{\partial p_j} = \frac{\partial u_i}{\partial G} \frac{\partial G}{\partial p_j} - \frac{\partial u_i}{\partial x_i} \left[p_i \frac{\partial G}{\partial p_j} + 2\alpha \left(\sum_{j=1}^n p_j - 1 \right) \right].$$

Since by assumption agent i is contributing a positive amount in the contribution stage of the game,

$$\frac{\partial u_i}{\partial G} - \frac{\partial u_i}{\partial x_i} p_i = 0,$$

and we are left with

$$\frac{\partial u_i}{\partial p_j} = -2\alpha \left(\frac{\partial u_i}{\partial x_i} \sum_{i=1}^n p_j - 1 \right) \neq 0.$$

Hence agent i can always change p_j in a way that will increase his utility. This completes the informal argument.

This argument shows in an intuitive way why the mechanism works. However, it is not a rigorous argument. The main problem is that in general we will not have an interior equilibrium in the contribution stage of the game. For arbitrary prices, it will typically be in the interest of some of the agents to free ride and contribute zero. However, this problem in the above argument can be patched up; there will always be *some* agent who makes a positive contribution to the public good and we only need show that he has an incentive to change his behavior if we are not at a Lindahl allocation.

In Varian (1989) I describe a related, but much simpler, mechanism that implements a Lindahl allocation in the case of quasilinear utility and two agents. In stage 1 each agent announces the rate at which they will subsidize the contributions of the other agent. In stage 2 each agent makes a contribution, paying for his own contribution at the subsidized rate, and paying the subsidy promised to the other agent. I show that the unique subgame perfect equilibrium to this game is the Lindahl allocation. Note that no penalty term is required in the case of two agents.

6. The proof that the mechanism works

We make the following assumptions about the agent's utility functions.

Assumption. The utility function $u_i(G, x_i)$ is a differentiable, quasiconcave function that is increasing in both arguments. The marginal-rate-of-substitution function, defined by

$$m_i(G, x_i) = \frac{\partial u_i / \partial G}{\partial u_i / \partial x_i},$$

is infinite when $G = 0$.

This assumption ensures that some agent will always want to contribute in the contribution stage, regardless of the prices set by the other agents. It can be relaxed significantly, but at the cost of complicating the argument.

The contribution stage

We begin by analyzing the second stage of the game. Given arbitrary prices $p_i > 0$, and arbitrary contributions by the other agents, $G_{-i} = \sum_{j \neq i} g_j$, agent i wants to solve the problem

$$\begin{aligned} & \max_{x_i, g_i} u_i(G_{-i} + g_i, x_i) \\ & \text{such that } x_i + p_i g_i = w_i - p_i G_{-i} - Q(p) \\ & g_i \geq 0. \end{aligned}$$

In interpreting the budget constraint, remember that each agent must pay an amount that depends on the sum of the contributions by the other agents, $p_i G_{-i}$, as well as a quadratic penalty $Q(p)$ depending on the announced prices.

If we add G_{-i} to both sides of these constraints and use the definition $G = G_{-i} + g_i$ we can rewrite this problem as

$$\begin{aligned} & \max_{x_i, G} u_i(G, x_i) \\ & \text{such that } x_i + p_i G = w_i - Q(p) \\ & G \geq G_{-i}. \end{aligned}$$

This is just like a standard two-good consumer maximization problem except for the inequality constraint. Although each agent i only gets to choose the level of his contribution g_i , he is effectively choosing level of the public good, since agent i can choose to contribute zero and get G_{-i} or to contribute a positive amount and get his preferred amount.

Let G_m be the maximum amount of the public good demanded by any of the agents at the prices (p_i) . Then I claim that G_m must be the Nash equilibrium amount of the public good. To prove this, simply note that G_m must solve each individual's utility maximization problem.

Note that the pattern of contributions among the agents who prefer to contribute G_m is arbitrary. To see this, suppose that agents i and j want the same amount of the public good, G_m , and that they are the only two agents making contributions to the public good. Let g_i and g_j denote these contributions; of course, $g_i + g_j = G_m$. However, i 's total payment is $p_i g_i + p_i G_{-i} = p_i (g_i + g_j) = p_i G_m$. Similarly agent j 's total payment is $p_j G_m$.

It follows that any reallocation of the contributions between i and j does not affect their total payments, which only depend on G_m and their personalized prices.

This has an important consequence which we will use below. If each individual's demand for the public good is a differentiable function of the price he faces, then the *equilibrium* amount of the public good will be left-differentiable function of the price faced by any contributor.

To see this, observe that there is no difficulty with differentiability when only one agent is contributing to the public good. The only difficulties arise when two or more agents each contribute. But we have just seen that when two agents contribute, they are indifferent about reallocations of the contributions between them—since the amount that they have to pay is independent of such reallocations. Suppose that we have an equilibrium in which agents i and j are both contributing. If we increase p_i a little bit, then agent i will want to stop contributing and free-ride on agent j . However, agent j will want to increase his contribution by exactly the amount that i was contributing. Hence the same amount of the public good will be provided, and the same payments will be made by each agent.

Suppose that we decrease p_i a little bit. Then agent i will want to increase his contribution to the public good by a small amount, and agent j will no longer contribute. Since we have assumed that agent i 's demand for the public good is a differentiable function of the price he faces, the equilibrium amount of the public good will only change by a small amount.

The price-setting stage

Consider now the price-setting stage of the game. We have already seen that each agent naming the Lindahl prices (p_i^e) is a Nash equilibrium.¹

So we only need to show that at any price vector other than the Lindahl prices, some agent can increase his utility. Let (p_i^*) the the set of prices that result from the announcements (p_{ij}^*) and let G^* be the Nash equilibrium amount of the public good in the second stage of the mechanism. We have to investigate three possible cases.

Case 1. $\sum_{k=1}^n p_k^* > 1$.

¹ More precisely, if each agent i names prices p_{ij} such that $\frac{1}{n-1} \sum_{i \neq j} p_{ij} = p_j^e$, then this is an equilibrium choice. All that matters is that the numbers p_{ij} named by the agents average out to the Lindahl prices. A small penalty based, say, on the variance of the p_{ij} 's would eliminate this degeneracy.

We have seen that at least one agent i must be contributing to the public good. Suppose that he changes his p_{ij}^* for some other agent j in such a way that the price facing agent j *decreases* by a small amount dp_j . This results in some change in the amount of the public good in the second stage dG , which may be zero.

The impact of this change on agent i 's utility is

$$du_i = \frac{\partial u_i}{\partial G} dG - \frac{\partial u_i}{\partial x_i} \left[p_i^* dG + 2\alpha \left(\sum_{k=1}^n p_k^* - 1 \right) \right] dp_j.$$

We can write this expression as

$$du_i = \left[\frac{\partial u_i}{\partial G} - \frac{\partial u_i}{\partial x_i} p_i^* \right] dG - 2\alpha \left(\frac{\partial u_i}{\partial x_i} \sum_{k=1}^n p_k^* - 1 \right) dp_j.$$

Since agent i is contributing a positive amount in the contribution stage of the game the bracketed expression vanishes, leaving us with

$$du_i = -2\alpha \frac{\partial u_i}{\partial x_i} \left(\sum_{k=1}^n p_k^* - 1 \right) dp_j > 0.$$

The inequality is due to the fact that dp_j is negative and all other terms are positive. Hence a small decrease in p_j^* must increase agent i 's utility.

Case 2. $\sum_{k=1}^n p_k^* < 1$.

Again, let i be an agent who is making a positive contribution, and now let him *increase* agent j 's price by dp_j . Let dG be the associated change in the equilibrium amount of the public good. Repeating the argument given for case 1, we are left with

$$du_i = -2\alpha \frac{\partial u_i}{\partial x_i} \left(\sum_{k=1}^n p_k^* - 1 \right) dp_j > 0.$$

Case 3. $\sum_{k=1}^n p_k^* = 1$.

Suppose first that everyone is contributing to the public good in the contribution stage of the game. Then the equilibrium amount of the public good, G^* , must satisfy the n first-order conditions

$$m_i(G^*, x_i^*) = p_i^*.$$

Summing these conditions gives us

$$\sum_{i=1}^n m_i(G^*, x_i^*) = 1.$$

But these are the conditions that characterize the Lindahl allocation, in which case were are done.

We are left with the case where some agent i is not contributing to the public good. If agent i is just on the verge of contributing, so that $m_i(G^*, x_i^*) = p_i^*$ the argument given above applies, so we may assume that $m_i(G^*, x_i^*) < p_i^*$.

Let agent i choose announcements (p_{ij}) such that the price for each agent j who is contributing to the public good increases by a small amount and thus the equilibrium level of the public good decreases by a small amount dG . The resulting change in utility for agent i is

$$du_i = \left[\frac{\partial u_i}{\partial G} - \frac{\partial u_i}{\partial x_i} p_i^* \right] dG > 0.$$

The inequality is due to the fact that the bracketed expression is negative since $m_i(G^*, x_i^*) < p_i^*$ and dG is negative by construction. Hence agent i can increase his utility by such a move.

This completes the argument: at any prices other than the Lindahl prices, there is some agent i that can increase his utility by naming different prices.

7. Related literature

There is a large literature on mechanisms that "solve the public goods problem." A solution generally means that one can exhibit a mechanism which has Nash equilibria that are efficient allocations. This literature up until 1979 is nicely surveyed by Groves (1979).

Clarke (1971) and Groves (1976) examine the public goods problem when all agents have quasilinear utility functions and exhibit a mechanism for which the dominant strategy of each agent is to report his true valuation of the public good. However, the resulting allocation is not in general Pareto efficient.

Groves and Ledyard (1977) present a game whose Nash equilibria are Pareto efficient but not Lindahl allocations. Hurwicz (1979) and Walker (1981) have described games

whose Nash equilibria are Lindahl allocations. The Hurwicz (1979) mechanism is in much the same spirit as our mechanism in that agents announce “prices” that in equilibrium turn out to be Lindahl prices. However, in the Hurwicz mechanism, each agent announces his *own* Lindahl price, not that of the other agents.

Moore and Repullo (1988) describe a mechanism whose subgame perfect equilibria implement an arbitrary choice correspondence—including the Lindahl correspondence. Moore and Repullo describe their construction in the case of quasilinear preferences, but indicate that it can be used for more general preferences.

In the Moore and Repullo mechanism, one agent announces his type and an other agent can challenge. If the second agent challenges, then we move to a subgame where agent 1 must choose between two allocations. The allocations in the subgame are chosen so that both agents will have an incentive to be truthful in the announcement and challenge stages.

This scheme has the advantage that it will work in very general environments. However, it has the disadvantage that one needs to specify the payoffs in advance for all the possible subgames. Although payoffs that satisfy the appropriate conditions will exist under general conditions, it may be quite complicated to actually specify them. In our mechanism, by contrast, the payoffs are very simple to specify.

8. A general treatment

We now consider a general treatment of the method. Suppose that there are n agents. Let x denote an allocation of k goods to the n agents. Each agent has a utility function defined over *allocations* denoted by $u_i(x)$. Since the utility functions are defined over allocations of the goods, each individual's utility can depend in an arbitrary way on other individuals' consumptions.

Let p_i be an nk -vector of personalized prices for agent i . Let x^* be an efficient allocation. We assume that the allocations x^* can be decentralized by a set of personalized prices, p_i^* in the sense that x^* is the solution to each agent's maximization problem

$$\begin{aligned} \max_x u_i(x) \\ \text{such that } p_i^* x \leq p_i^* \omega_i. \end{aligned} \tag{3}$$

Here ω_i is agent i 's initial endowment of goods. This assumption essentially requires that preferences be convex.

Given any set of personalized prices (p_i), denote the solution to agent i 's maximization problem (3) by x_i . Note that x_i is an *allocation*, not a consumption bundle. For simplicity, assume that x_i is uniquely determined.

For an arbitrary set of personalized prices, the set of proposed allocations will not necessarily be feasible. Even if everyone proposes the same feasible allocation, it will not necessarily be efficient. Let $F(p)$ denote a measure of how far the allocations fail to be feasible, and let $E(p)$ denote a measure of by how much the allocation fails to be efficient. These functions should be nonnegative functions that equal zero if and only if the allocations are feasible or efficient, respectively, and are positive otherwise. The quadratic forms used earlier are a natural choice.

We can now state the appropriate mechanism to solve our resource allocation problem.

Announcement stage. Each agent i announces a personalized price vector p_{ij} for each other agent j . The price announcements are averaged to construct the personalized prices facing agent j .

Consumption stage. Each agent chooses x_i to maximize his utility, given his personalized prices p_i . However, each agent must also pay a penalty of $\alpha[E(p) + F(p)]$ where p is the vector of all of the personalized prices and α is a strictly positive number.

Once the prices have been determined agent i 's maximization problem is:

$$\begin{aligned} \max_x u_i(x) \\ \text{such that } p_i x \leq p_i \omega_i - \alpha[E(p) + F(p)]. \end{aligned} \tag{4}$$

I claim that the subgame perfect equilibria of this game are efficient allocations.

The proof is a simple variation on the earlier arguments. Let x^* be an efficient allocation. Then, by hypothesis, this allocation is supported by a set of personalized prices (p_i^*). At the equilibrium set of (p_i^*) the penalty term $E(p^*) + F(p^*)$ equals zero. Changing his price announcements for the other agents doesn't affect agent i 's price, and can only increase the value of the penalty term. Hence (p_i^*, x^*) is a subgame perfect equilibrium for the game.

Consider any other set of prices that do not minimize $E(p) + F(p)$. Then some agent must have an incentive to adjust his price announcement in a way that decreases the value of his penalty term. Hence, no other set of prices can be an equilibrium of the game. This completes the argument.

Discussion of the general result

The logic behind the general mechanism is simple: we express our desired allocation as the zero of some penalty function, and then give each agent an incentive to minimize that penalty function. Stated in this way, the result is trivial. However, as we have seen earlier, this general principle can be tailored to specific resource allocation problems in a way that makes the mechanism much more useful.

For example, in the case of a simple externality discussed earlier, one firm controls the level of the externality directly. In this case there is no problem with feasibility, and we only need to ensure that the efficiency condition is met; i.e., that the marginal cost of the externality be equated across the firms.

In the case of the public goods problem, we can use the idea of a Lindahl allocation to ensure that the feasibility condition is automatically satisfied. I expect that this can be done in a large number of cases when the resource allocation problem has sufficient structure.

9. Summary

We have exhibited a class of mechanisms that implement efficient outcomes in classical environments involving externalities and public goods. Several questions remain for future research. Of course, it would be nice to have a mechanism that works when each agent only knows his own type. However, it seems unlikely that such a mechanism exists.

First, it would be nice to have a simple mechanism for the public goods case that does not require simultaneous moves. Second, it would be nice to find a mechanism that works when agents have incomplete information about the types of the other agents. Third, it would be useful to examine applications of this method to other problems of resource allocation. Finally, it would be very interesting to see how well this mechanism performs in real life. I suspect that some progress will be possible on each of these fronts.

References

- Bergstrom, T., L. Blume, and H. Varian (1986), "On the Private Provision of Public Goods," *Journal of Public Economics*, 29, 25-49.
- Clarke, E. (1971), "Multipart Pricing of Public Goods," *Public Choice*, 11, 17-33.
- Groves, T. (1976), "Information, Incentives, and the Internalization of Production Externalities," in Lin, S. (ed.) *Theory and Measurement of Economic Externalities*, Academic Press, New York.
- Groves, T. and J. Ledyard (1977), "Optimal Allocations of Public Goods: A Solution to the 'Free Rider Problem'," *Econometrica*, 45, 783-809.
- Groves, T. (1979), "Efficient Collective Choice when Compensation is Possible," *Review of Economic Studies*, 227-241.
- Laffont, J.-J. and E. Maskin (1982), "The Theory of Incentives: An Overview," in *Advances in Economic Theory: Invited Papers for the Fourth World Congress of the Econometric Society, 1980*, ed. by W. Hildenbrand, Cambridge University Press, Cambridge, 31-94.
- Maskin, E. (1985), "The Theory of Implementation in Nash Equilibrium," in *Social Goals and Social Organization*, ed. by L. Hurwicz, D. Schmeidler, and H. Sonnenschein, Cambridge University Press, Cambridge, 173-204.
- Matsushima, Hitoshi (1988), "A New Approach to the Implementation Problem," *Journal of Economic Theory*, 45, 128-144.
- Palfrey, Thomas and Sanjay Srivastava (1989), "Implementation with Incomplete Information in Exchange Economies," *Econometrica*, 57, 115-134.
- Hurwicz, L. (1979), "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," *Review of Economic Studies*, 217-225.
- Moore, J. and R. Repullo (1988), "Subgame Perfect Implementation," *Econometrica*, 56, 1191-1220.
- Varian, H. (1989), "Sequential Provision of Public Goods," University of Michigan CREST Working Paper.
- Walker, M. (1981), "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations," *Econometrica*, 48, 56-73.

Working Papers of the Department of Economics

Published since 1989

89/370

B. BENSARD/R.J. GARY-BOBO/
S. FEDERBUSCH
The Strategic Aspects of Profit Sharing in the
Industry

89/374

Francisco S. TORRES
Small Countries and Exogenous Policy Shocks

89/375

Renzo DAVIDDI
Rouble Convertibility: A Realistic Target

89/377

Elettra AGLIARDI
On the Robustness of Contestability Theory

89/378

Stephen MARTIN
The Welfare Consequences of Transaction Costs
in Financial Markets

89/381

Susan SENIOR NELLO
Recent Developments in Relations Between the
EC and Eastern Europe

89/382

Jean GABSZEWICZ/ Paolo GARELLA/
Charles NOLLET
Spatial Price Competition With Uninformed
Buyers

89/383

Benedetto GUI
Beneficiary and Dominant Roles in
Organizations: The Case of Nonprofits

89/384

Agustín MARAVALL/ Daniel PEÑA
Missing Observations, Additive Outliers and
Inverse Autocorrelation Function

89/385

Stephen MARTIN
Product Differentiation and Market Performance
in Oligopoly

89/386

Dalia MARIN
Is the Export-Led Growth Hypothesis Valid for
Industrialized Countries?

89/387

Stephen MARTIN
Modeling Oligopolistic Interaction

89/388

Jean-Claude CHOURAQUI
The Conduct of Monetary Policy: What have we
Learned From Recent Experience

89/390

Corrado BENASSI
Imperfect Information and Financial Markets: A
General Equilibrium Model

89/394

Serge-Christophe KOLM
Adequacy, Equity and Fundamental Dominance:
Unanimous and Comparable Allocations in
Rational Social Choice, with Applications to
Marriage and Wages

89/395

Daniel HEYMANN/ Axel LEIJONHUFVUD
On the Use of Currency Reform in Inflation
Stabilization

89/400

Robert J. GARY-BOBO
On the Existence of Equilibrium Configurations
in a Class of Asymmetric Market Entry Games

89/402

Stephen MARTIN
Direct Foreign Investment in The United States

89/413

Francisco S. TORRES
Portugal, the EMS and 1992: Stabilization and
Liberalization

89/416

Joerg MAYER
Reserve Switches and Exchange-Rate Variability:
The Presumed Inherent Instability of the
Multiple Reserve-Currency System

89/417

José P. ESPERANÇA/ Neil KAY
Foreign Direct Investment and Competition in
the Advertising Sector: The Italian Case

89/418

Luigi BRIGHI/ Mario FORNI
Aggregation Across Agents in Demand Systems

89/420

Corrado BENASSI

A Competitive Model of Credit Intermediation

89/422

Marcus MILLER/ Mark SALMON

When does Coordination pay?

89/423

Marcus MILLER/ Mark SALMON/

Alan SUTHERLAND

Time Consistency, Discounting and the Returns to Cooperation

89/424

Frank CRITCHLEY/ Paul MARRIOTT/

Mark SALMON

On the Differential Geometry of the Wald Test with Nonlinear Restrictions

89/425

Peter J. HAMMOND

On the Impossibility of Perfect Capital Markets

89/426

Peter J. HAMMOND

Perfect Option Markets in Economies with Adverse Selection

89/427

Peter J. HAMMOND

Irreducibility, Resource Relatedness, and Survival with Individual Non-Convexities

ECO No. 90/1*

Tamer BAŞAR and Mark SALMON

Credibility and the Value of Information

Transmission in a Model of Monetary Policy and Inflation

ECO No. 90/2

Horst UNGERER

The EMS – The First Ten Years

Policies – Developments – Evolution

ECO No. 90/3

Peter J. HAMMOND

Interpersonal Comparisons of Utility: Why and how they are and should be made

ECO No. 90/4

Peter J. HAMMOND

A Revelation Principle for (Boundedly) Bayesian Rationalizable Strategies

ECO No. 90/5

Peter J. HAMMOND

Independence of Irrelevant Interpersonal Comparisons

ECO No. 90/6

Hal R. VARIAN

A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed

ECO No. 90/7

Hal R. VARIAN

Sequential Provision of Public Goods

ECO No. 90/8

T. BRIANZA, L. PHLIPS and J.F. RICHARD

Futures Markets, Speculation and Monopoly Pricing

* Please note: As from January 1990, the EUI Working Papers Series is divided into six sub-series, each series will be numbered individually (e.g. EUI Working Paper LAW No 90/1).



EUI WORKING PAPERS

**EUI Working Papers are published and distributed by the
European University Institute, Florence**

**Copies can be obtained free of charge – depending on the availability of
stocks – from:**

**The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy**

Please use order form overleaf

Publications of the European University Institute

To The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy

From Name

Address

.....
.....
.....
.....

- ☐ Please send me a complete list of EUI Working Papers
- ☐ Please send me a complete list of EUI book publications
- ☐ Please send me the EUI brochure Academic Year 1990/91

Please send me the following EUI Working Paper(s):

No, Author

Title:

No, Author

Title:

No, Author

Title:

No, Author

Title:

Date

Signature



89/384

Agustín MARAVALL/

Daniel PEÑA

Missing Observations, Additive
Outliers and Inverse Auto-
correlation Function

89/385

Stephen MARTIN

Product Differentiation and
Market Performance in Oligopoly

89/386

Dalia MARIN

Is the Export-Led Growth
Hypothesis Valid for
Industrialized Countries?

89/387

Stephen MARTIN

Modeling Oligopolistic Interaction

89/388

Jean-Claude CHOURAQUI

The Conduct of Monetary Policy:
What have we Learned From
Recent Experience

89/389

Léonce BEKEMANS

Economics in Culture vs. Culture
in Economics

89/390

Corrado BENASSI

Imperfect Information and
Financial Markets: A General
Equilibrium Model

89/391

Patrick DEL DUCA

Italian Judicial Activism in Light
of French and American
Doctrines of Judicial Review and
Administrative Decisionmaking:
The Case of Air Pollution

89/392

Dieter ZIEGLER

The Bank of England in the
Provinces: The Case of the
Leicester Branch Closing, 1872

89/393

Gunther TEUBNER

How the Law Thinks:
Toward a Constructivist
Epistemology of Law

89/394

Serge-Christophe KOLM

Adequacy, Equity and
Fundamental Dominance:
Unanimous and Comparable
Allocations in Rational Social
Choice, with Applications to
Marriage and Wages

89/395

Daniel HEYMANN/

Axel LEIJONHUFVUD

On the Use of Currency Reform
in Inflation Stabilization

89/396

Gisela BOCK

Challenging Dichotomies:
Theoretical and Historical
Perspectives on Women's Studies
in the Humanities and Social
Sciences

89/397

Giovanna C. CIFOLETTI

Quaestio sive aequatio:

la nozione di problema nelle
Regulae

89/398

Michela NACCI

L'équilibre difficile. Georges
Friedmann avant la sociologie du
travail

89/399

Bruno WANROOIJ
Zefthe Akaira, o delle identità
smarrite

89/400

Robert J. GARY-BOBO
On the Existence of Equilibrium
Configurations in a Class of
Asymmetric Market Entry Games

89/401

Federico ROMERO
The US and Western Europe:
A Comparative Discussion of
Labor Movements in the Postwar
Economy

89/402

Stephen MARTIN
Direct Foreign Investment in the
United States

89/403

Christine LAMARRE
La vie des enfants et des vieillards
assistés à Dijon au 18^e siècle

89/404

Christian JOERGES
Product liability and product
safety in the European
Community

89/405

Giandomenico MAJONE
Regulating Europe:
Problems and Prospects

89/406

Fabio SDOGATI
Exchange Rate Fluctuations and
the Patterns of International
Trade: A Study of the Flow of
Trade from Newly Industrialized
Countries to the European
Community at the Industry Level

89/407

Angela LIBERATORE
EC Environmental Research and
EC Environmental Policy:
A study in the utilization of
knowledge for regulatory
purposes

89/408

J.-Matthias Graf von der
SCHULENBURG
Regulation and Deregulation of
Insurance Markets in the Federal
Republic of Germany

89/409

Greg KASER
Acceptable Nuclear Risk: Some
Examples from Europe

89/410

Léonce BEKEMANS/Manfred
GLAGOW/Jeremy MOON
Beyond Market and State
Alternative Approaches to
Meeting Societal Demands

89/411

Erich KAUFER
The Regulation of Drug
Development: In Search of a
Common European Approach

89/412

Gianna GIANNELLI/
Gøsta ESPING-ANDERSEN
Labor Costs and Employment in
the Service Economy

89/413

Francisco S. TORRES
Portugal, the EMS and 1992:
Stabilization and Liberalization

89/414

Gøsta ESPING-ANDERSEN/
Harald SONNBERGER
The Demographics of Age in
Labor Market Management

89/415

Fritz von NORDHEIM NIELSEN
The Scandinavian Model:
Reformist Road to Socialism or
Dead End Street?

89/416

Joerg MAYER
Reserve Switches and Exchange-
Rate Variability: The Presumed
Inherent Instability of the Multiple
Reserve-Currency System

89/417

José P. ESPERANÇA/Neil KAY
Foreign Direct Investment and
Competition in the Advertising
Sector: The Italian Case

89/418

Luigi BRIGHI/Mario FORNI
Aggregation Across Agents in
Demand Systems

89/419

Hans Ulrich JESSURUN
d'OLIVEIRA
Nationality and Apartheid: Some
Reflections on the Use of
Nationality Law as a Weapon
against Violation of Fundamental
Rights

89/420

Corrado BENASSI
A Competitive Model of Credit
Intermediation

89/421

Ester STEVERS
Telecommunications Regulation in
the European Community: The
Commission of the European
Communities as Regulatory Actor

89/422

Marcus MILLER/Mark SALMON
When does Coordination pay?

89/423

Marcus MILLER/Mark
SALMON/
Alan SUTHERLAND
Time Consistency, Discounting
and the Returns to Cooperation

89/424

Frank CRITCHLEY/Paul
MARRIOTT/Mark SALMON
On the Differential Geometry of
the Wald Test with Nonlinear
Restrictions

89/425

Peter J. HAMMOND
On the Impossibility of Perfect
Capital Markets

89/426

Peter J. HAMMOND

**Perfected Option Markets in
Economies with Adverse Selection**

89/427

Peter J. HAMMOND

**Irreducibility, Resource
Relatedness, and Survival with
Individual Non-Convexities**

89/428

Joanna GOYDER

**"Business Format" Franchising
and EEC Competition Law**

EUI Working Papers as from 1990

Please note: As from January 1990, the EUI Working Papers Series is divided into six sub-series, each series will be numbered individually (e.g. EUI Working Paper LAW No 90/1).

Working Papers in Law

LAW No 90/1

David NELKEN

The Truth about Law's Truth

LAW No 90/2

Antonio CASSESE/Andrew

CLAPHAM/Joseph H.H. WEILER

1992 – What are our Rights?

Agenda for a Human Rights

Action Plan

Working Papers of the European Policy Unit

EPU No 90/1

Renaud DEHOUSSE/Joseph H.H.

WEILER

EPC and the Single Act:

From Soft Law to Hard Law?

Working Papers in Economics

ECO No. 90/1

Tamer BAŞAR/Mark SALMON
Credibility and the Value of
Information Transmission in a
Model of Monetary Policy and
Inflation

ECO No. 90/8

T. BRIANZA/L. PHILIPS/J.-F.
RICHARD
Futures Markets, Speculation and
Monopoly Pricing

ECO No. 90/2

Horst UNGERER
The EMS – The First Ten Years
Policies – Developments –
Evolution

ECO No. 90/3

Peter J. HAMMOND
Interpersonal Comparisons of
Utility: Why and how they are
and should be made

ECO No. 90/4

Peter J. HAMMOND
A Revelation Principle for
(Boundedly) Bayesian
Rationalizable Strategies

ECO No. 90/5

Peter J. HAMMOND
Independence of Irrelevant
Interpersonal Comparisons

ECO No. 90/6

Hal R. VARIAN
A Solution to the Problem of
Externalities and Public Goods
when Agents are Well-Informed

ECO No. 90/7

Hal R. VARIAN
Sequential Provision of Public
Goods

